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COMPUTATION OF GAS-DYNAMIC PARAMETERS AND HEAT TRANSFER IN SUPERSONIC TURBULENT SEPARATED FLOWS NEAR BACKWARD-FACING STEPS

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Computation results of plane turbulent flows in the vicinity of backward-facing steps with leeward-face angles $\beta = 8$, 25, and 45° for Mach numbers $M_\infty = 3$ and 4 are presented. The averaged Navier–Stokes equations supplemented by the Wilcox model of turbulence are used as a mathematical model. The boundary-layer equations were also used for the case of an attached flow ($\beta = 8°$). The computed and experimental distributions of surface pressure and skin friction, the velocity and pressure fields, and the heat-transfer coefficients are compared.

Introduction. Shock wave–boundary layer interaction is one of the classical problems of gas dynamics. Calculating the parameters of the boundary-layer separation resulting from this interaction is important both for practical applications and for understanding the physics of these processes. Supersonic turbulent separated flows are studied numerically in many papers. A comparison of calculation results obtained using various algebraic and differential models of turbulence with experimental data for such flows is given Marvin and Coakley [1]. It is shown that the use of differential models is preferable for separated flows, and modification of these models is necessary to take into account compressibility and improve the accuracy of heat-transfer calculations. Knight [2] made a review of the papers on numerical simulation of compressible turbulent flows and concluded that the agreement of calculated and experimental results is reached by using turbulence models developed specially for this class of flows. Knight and Degrez [3] evaluated the possibility of simulating 2D and 3D interactions of shock waves and boundary layers. It was shown that the computed distribution of static pressure is in good agreement with the experiment, whereas the accuracy of surface-friction and heat-transfer predictions based on the available averaged Navier–Stokes solvers is unsatisfactory. It was recommended in [3] to use large eddy simulation for these flows, but this approach requires powerful computers and significant computer time. Thus, the most suitable method for closing the averaged Navier–Stokes equations in engineering calculations is still the use of differential models of turbulence.

The present paper is a continuation of studies dealing with numerical simulation of supersonic turbulent separated flows. Borisov and Fedorova [4] analyzed the flow around forward-facing steps with different slopes of the leeward face, proposed a mathematical model and a numerical algorithm. The results obtained using a scheme of splitting inviscid fluxes with respect to physical processes showed that the computation results depend on the algorithm resolution. The use of first-order relations for approximation of convective terms leads to smearing of the separation shock, significant underestimation of the separation-region scale, and consequently, to distortion of all flow parameters.

Borisov et al. [5] studied numerically the evolution of supersonic turbulent separated flows near 2D forward-facing steps with variation of the leeward-face slope from 8 to 90°. An analysis of experimental and

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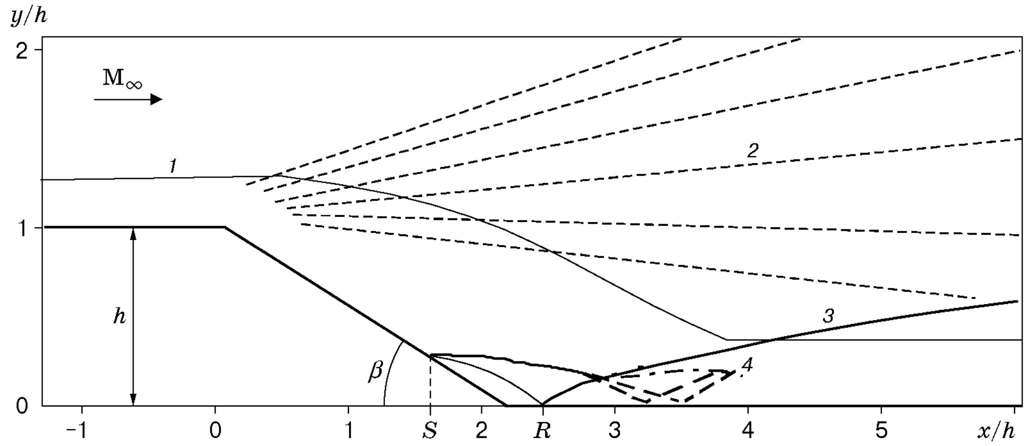

Fig. 1. Experimental scheme of the flow: boundary-layer edge (1), expansion fan behind the expansion corner (2), λ-configuration of shock waves formed by separated and reattachment shocks (3), and secondary expansion fan emanating from the triple point of the λ-configuration (4).

numerical data showed that, despite the unsteady effects typical of these flows and manifested most strongly in the vicinity of separation and reattachment points, mathematical simulation on the basis of the averaged Navier–Stokes equations and the k–ω model of turbulence gives a good prediction of the separation-region scale, the gas-dynamic flow structure, and the surface-pressure and skin-friction distributions.

In [6], the properties of three TVD schemes that employ different methods of splitting of the inviscid-flux vector were studied on the problem of modeling of a turbulent separated flow near a rectangular forward-facing step for a Mach number $M_\infty = 3$. It was shown that the properties of numerical solutions depend strongly on the algorithm resolution. The dependence of results on the turbulence-model parameter governing the balance of dissipation processes and generation of turbulent kinetic energy was studied.

The computation results for plane turbulent flows near steps with leeward-face angles $\beta = 8$, 25, and 45° are described in the present paper. The objective of the paper is to verify the efficiency of the numerical algorithm with a changed sequence of interaction, where the boundary layer interacts first with the expansion fan and then with the shock wave. The computations were performed within a wide range of Mach and Reynolds numbers. As was noted above, the most difficult procedure is the heat-transfer prediction for turbulent flows; therefore, special attention was given to this problem. In addition, the boundary-layer equations supplemented by the Wilcox turbulence model were used in the case of an attached flow around the step with an angle of 8° to check the applicability of the simplified approach.

1. Mathematical Model. The averaged Navier–Stokes equations supplemented by the Wilcox k–ω model of turbulence [7] were used as a mathematical model. For temporal approximation, an implicit scheme of splitting with respect to spatial variables was used, which was implemented by scalar sweeps. The derivatives of inviscid flows were approximated using several TVD schemes based on the flux-vector splitting proposed by van Leer [8] and the scheme of splitting with respect to physical processes [9]. The numerical algorithm was described in detail and studied in [4, 6]. By the example of computation of the flow around a rectangular forward-facing step, it was shown in [6] that the scheme based on splitting the inviscid-flux vector with respect to physical processes has a better resolution; therefore, it was chosen as the basic scheme for computations.

2. Computation Results. The computations were performed under the conditions of physical experiments of [10], which contain all the results necessary for comparison: the pressure and skin friction distributions over the body surface, the pressure and velocity fields within the entire flow region examined, and the distribution of heat-transfer coefficients.

The flow pattern in the vicinity of backward-facing steps depends on the slope of its leeward face β and the Mach number. For $\beta = 8°$ the flow remains attached, and for $\beta = 25$ and 45° separation appears, which

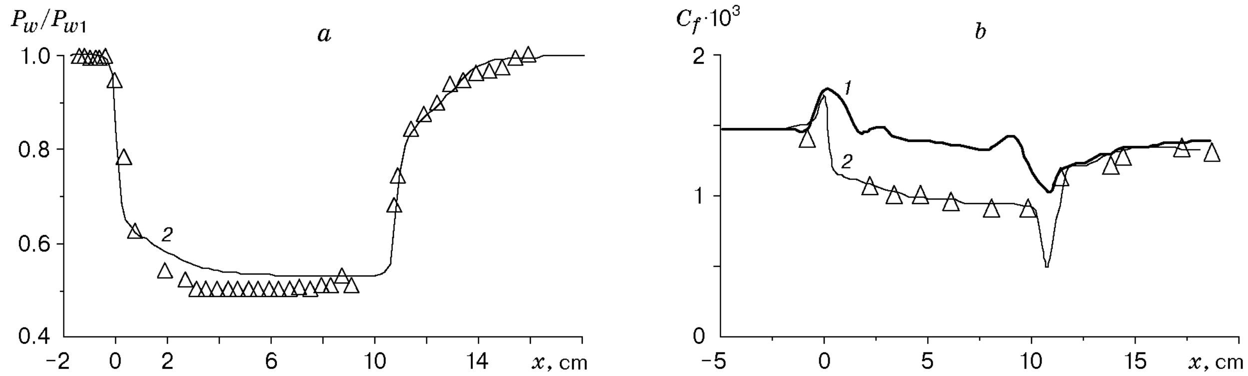

Fig. 2. Distributions of surface pressure (a) and skin friction (b) for $\beta = 8°$, $M_\infty = 3$, and $h = 15$ mm: curves 1 and 2 refer to boundary-layer and Navier–Stokes calculations, respectively; the points are experimental data.

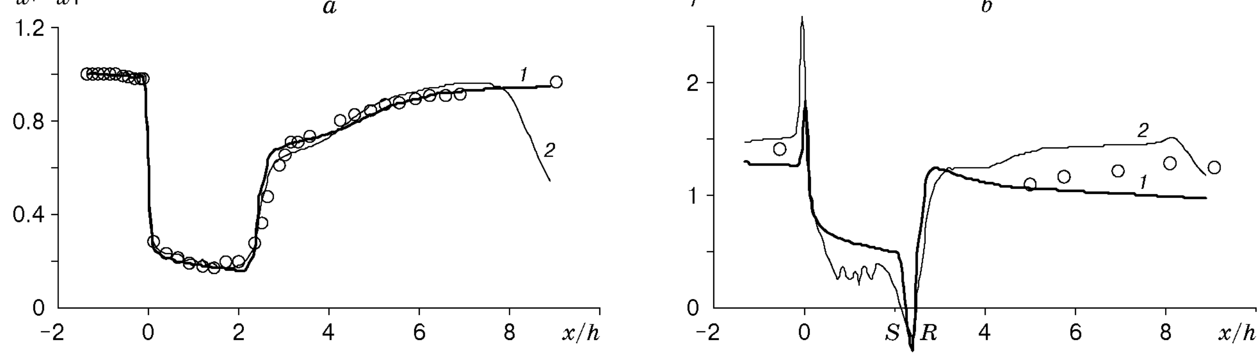

Fig. 3. Distributions of surface pressure (a) and skin friction (b) for $\beta = 25°$, $M_\infty = 3$, and $h = 15$ mm: curves 1 and 2 refer to computations by the schemes of [8] and [9], respectively; the points are experimental data.

is caused by interaction of the boundary layer with a shock wave in the compression corner, which is stronger than in the case $\beta = 8°$. Figure 1 shows the experimental flow pattern for a step of height $h = 15$ mm with a leeward-face angle $\beta = 25°$ for $M_\infty = 4$. In particular, the separation region limited by the separation (S) and reattachment (R) points is shown. The presence of a secondary expansion fan 4 depends on the relationship of parameters on the separated, reattachment, and main shock waves and on the position of the triple point of the λ-configuration.

Computation of Gas-Dynamic Parameters. Figure 2 shows the computed distributions of surface pressure P_w/P_{w1} and skin friction C_f for a backward-facing step of height $h = 15$ mm with an angle $\beta = 8°$ for $M_\infty = 3$. The computations were performed using two approaches: Navier–Stokes and boundary-layer equations. In both cases, the Wilcox k–ω model of turbulence was used to close the system.

In boundary-layer equations, the experimental pressure distribution along the surface was set as an external action. An analysis of results showed that the integral parameters are well predicted by both methods, and the skin friction on the inclined face is overpredicted in the boundary-layer computations. This may be attributed to the fact that the boundary-layer model has no equation for y-momentum, which leads to incorrect results in those regions where the transverse momentum appears.

The computation results presented below were obtained within the framework of the full Navier–Stokes equations.

Figure 3 shows the surface-pressure and skin-friction distributions for $\beta = 25°$ and $M_\infty = 3$. The computations were performed using the van Leer schemes [8] and splitting with respect to physical processes [9] for approximation of inviscid fluxes. In contrast to the previous case ($\beta = 8°$), a small separation region appears in the vicinity of the compression corner. The separation may be observed in the skin-friction

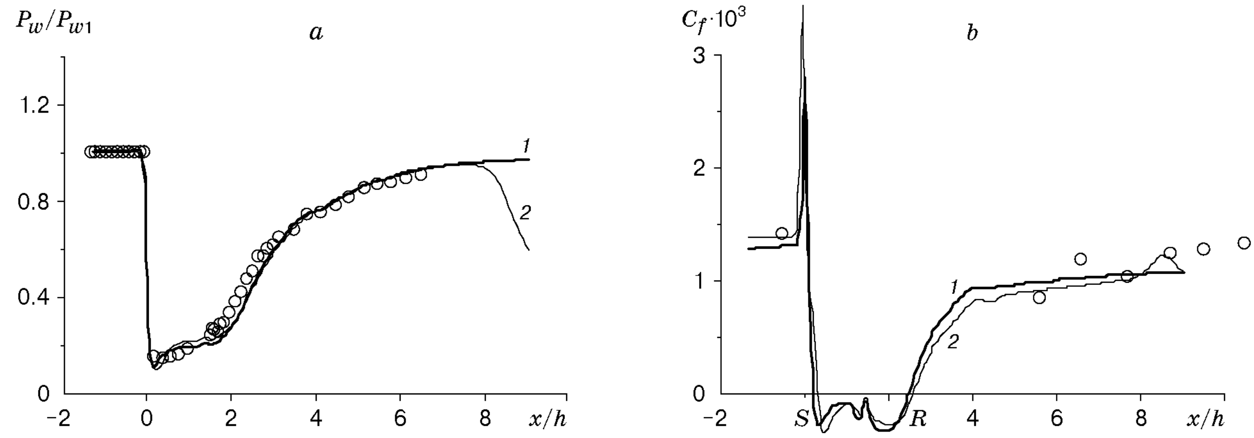

Fig. 4. Distributions of surface pressure (a) and skin friction (b) for $\beta = 45°$, $M_\infty = 3$, and $h = 15$ mm: curves 1 and 2 refer to computations by the schemes of [8] and [9], respectively; the points are experimental data.

curve for $x/h \approx 2$. The experimental positions of the separation point S and reattachment point R are also shown in Fig. 3. The computed results are in agreement with experimental data in the region of shock wave–boundary layer interaction for both splitting schemes. Note that the distinctive feature in the scheme of [9] is the presence of an intense expansion fan behind the compression corner, which is absent in the computations with the inviscid-flux splitting scheme [8]. The existence of this expansion fan has not been confirmed experimentally; some considerations that evidence indirectly its presence are given below.

The source of the expansion fan in computations is the triple point formed by the main shock wave, separated shock, and centered compression waves. The set of solutions in the vicinity of the triple point is rather complicated even for a purely inviscid flow (see, e.g., [11]) and contains a solution with an expansion fan. This solution is observed near the triple point of the λ-configuration formed above the separation region, which is confirmed by the physical experiment (see Fig. 1). The conditions of occurrence of this or that solution in the vicinity of the triple point are determined by numerous factors, including effective viscosity in the flow region downstream of the reattachment point. If the computed and experimental parameters of turbulence near the triple point are different, conditions that are not observed in the experiment arise in computations, which is responsible for the difference in the computed and experimental data within the entire region. A possible reason for the significant differences is also the properties of the difference scheme that selects this solution among the set of admissible solutions.

Figure 4 shows the distributions of surface pressure and skin friction for $\beta = 45°$ and $M_\infty = 3$. The formation of a vast separation region is typical of a backward-facing step with $\beta = 45°$. The separation point tends to the limiting position immediately behind the apex of the expansion corner. The results computed with the use of both splitting schemes are in good agreement with experimental data in the region of interaction of the boundary layer with expansion and shock waves. It is seen in Fig. 4 that the separation-region length obtained in computations by the schemes of [8] and [9] is in good agreement with the experimental value. A typical plateau is observed in Fig. 4a in the region $x/h = 1$–2, which is caused by the λ-configuration of the separation and reattachment shock waves. As in the previous case, the difference between the computations by different schemes is caused by the presence of intense expansion waves induced by the triple point and incident onto the surface in the computations by the scheme of [9].

Figure 5 shows the profiles of the streamwise velocity U/U_e in various cross sections for $\beta = 45°$ and $M_\infty = 3$. The values $x/h = 0.9$ and 1.33 refer to the separation region, and $x/h = 7.7$ refers to the reattached boundary layer. The velocity profiles are in good agreement with experimental data behind the reattachment point but differ significantly in the separation region. It is seen from Fig. 5 that the velocity maximum in the reverse flow is lower in computations by both schemes than in the experiment. As is noted in [12], the development of separation during interaction of the shock wave with the boundary layer disturbed by

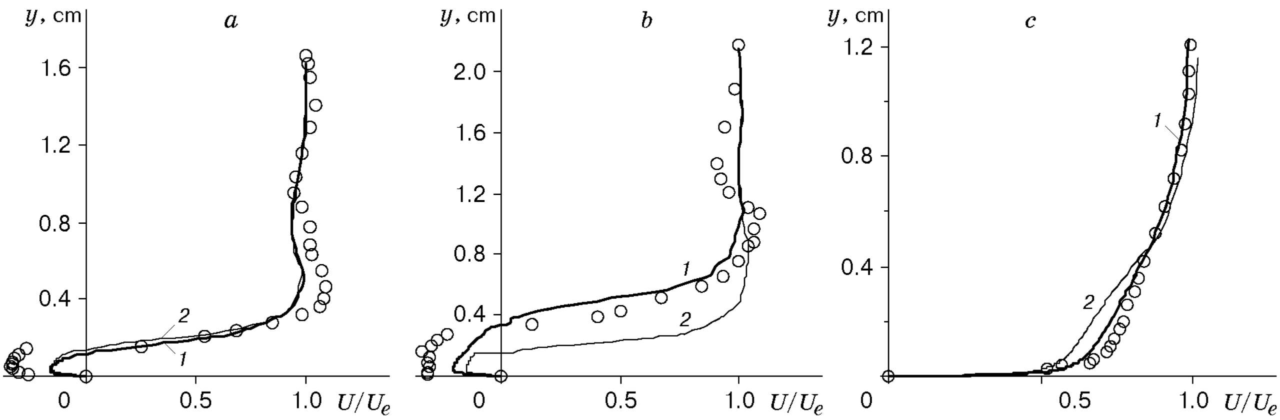

Fig. 5. Profiles of the streamwise velocity in various cross sections for $\beta = 45°$, $M_\infty = 3$, $h = 15$ mm, and $x/h = 0.9$ (a), 1.33 (b), and 7.7 (c); curves 1 and 2 refer to computations by the schemes of [8] and [9], respectively; the points are experimental data.

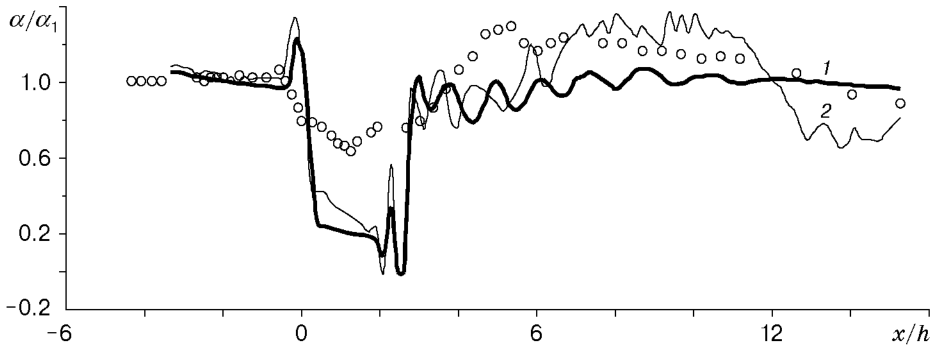

Fig. 6. Distribution of the heat-transfer coefficient for $\beta = 25°$, $M_\infty = 3$, and $h = 6$ mm: curves 1 and 2 refer to computations by the schemes of [8] and [9], respectively; the points are experimental data.

expansion waves is affected by two processes. On the one hand, the increasing fullness of the velocity profile, which is caused by flow acceleration in expansion waves, prevents the separation. On the other hand, the suppression of turbulent oscillations in expansion waves facilitates separation. It follows from the analysis of numerical data that the turbulence model, apparently, does not reproduce completely the decrease in oscillations in expansion waves on the inclined face, which leads to a lower absolute value of velocity in the reverse flow.

Heat-Transfer Computation. Computation of the heat-transfer intensity is one of the most complicated problems of mathematical simulation of turbulent flows. Even if the numerical algorithm provides an adequate prediction of the parameters in an actual flow, such as the static pressure, velocity, temperature, and skin friction, this does not mean that the heat-transfer coefficients are predicted correctly. The reason is that the level of flow turbulence has a greater effect on heat transfer than on other parameters. The ability of the numerical model and turbulence model in computing the heat-transfer intensity depends on their ability of correctly describing the processes of generation and dissipation of the turbulent kinetic energy. Thus, heat-transfer computations verify the applicability of the turbulence model for conditions of a nonequilibrium boundary layer disturbed by shock waves and expansion waves. In the present work, we compare the computed heat-transfer coefficient with experimental data for the flow around a backward-facing step with $\beta = 25°$. Heat transfer was experimentally studied for steps of height $h = 6$ mm. Other gas-dynamic parameters were not measured.

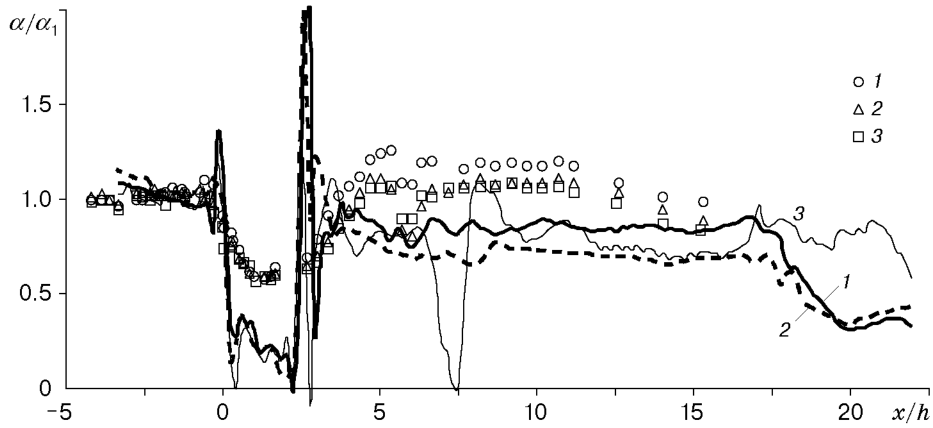

Fig. 7. Distribution of the heat-transfer coefficient for $\beta = 25°$, $M_\infty = 3$, $h = 6$ mm, and $\mathrm{Re}_1 = 40 \cdot 10^6$ (points 1), $58 \cdot 10^6$ (points 2), and $89 \cdot 10^6$ m^{-1} (points 3); the curves are calculated dependences and the points are experimental data.

Figure 6 shows the distribution of the normalized heat-transfer coefficient for $\beta = 25°$ and $M_\infty = 3$. Here $\alpha = (\partial T/\partial n)/(T_w - T_{ad})$ (α_1 is the heat-transfer coefficient prior to interaction, T_w is the temperature of the hot wall, and T_{ad} is the temperature of the adiabatic wall). It is seen from Fig. 6 that the level of heat-transfer intensity on the inclined face ($x/h = 0$–2) is significantly underpredicted in the computations by the schemes of [8, 9]. As is noted above, the reason may be the mathematical model that inadequately reproduces the process of suppression of turbulent oscillations in expansion waves.

We note that the heat-transfer coefficients calculated by both splitting schemes coincide in the region $x/h = 0$–6. The behavior of the calculated curve obtained using the scheme of [9] demonstrates an increase in heat-transfer intensity in the region of compression waves and a subsequent decrease at the place of incidence of the secondary expansion waves. A decrease in the heat-transfer coefficient is also observed in this region in the experiment. This behavior of $\alpha(x)$ gives indirect evidence of the existence of secondary expansion waves incident onto the plate surface rather far from the interaction region. However, the decrease in heat-transfer intensity in the experiment may be caused by some other reasons (for example, by three-dimensional effects). An additional study is needed to clarify the reasons for the decrease in heat-transfer intensity in this region and the applicability of the flow scheme proposed.

Figure 7 shows the distribution of the heat-transfer coefficient for different Reynolds numbers. The computations are performed using the scheme of [9]. We note that the computed dependence of heat-transfer intensity as a function of the Reynolds number is in agreement with experimental data. In particular, for $\mathrm{Re}_1 = 89 \cdot 10^6$ m^{-1}, the computation reflects the experimentally registered decrease in heat-transfer intensity in the region $x/h = 6.0$–7.5, which is caused by expansion waves emanating from the triple point of the λ-configuration (see Fig. 1). In this case, the computation predicts a greater decrease in heat-transfer intensity as compared to that registered in the experiment. The decrease in the heat-transfer level for $x/h > 10$, which is observed in the experiment for three values of Re_1, may be related to the action of secondary expansion waves, as is shown by the computations. However, these waves are located more downstream here, and their effect on decreasing the heat-transfer level is more significant than in the experiment.

It is shown by numerical simulation of the flow around rectangular forward-facing steps [6] that the relationship between the processes of generation and dissipation of turbulence may be controlled by the turbulence-model parameter that limits from below the minimum of specific dissipation of the turbulent kinetic energy ω. The use of the limiter ω_0 allows one to control the generation of turbulence in the shock wave and significantly improve the accuracy of computation of the separation-region length. Nevertheless, the use of this limiter is little effective in the flow around backward-facing steps, where the order of interaction of the boundary layer with disturbances is different. A possible reason is that this parameter "controls" the magnitude of turbulent oscillations in the external (wake) region of the boundary layer. As is noted in [13], exactly the values of turbulent viscosity in the external part of the boundary layer determine the parameters of large-scale separation. The limiter affects the internal part of the boundary layer and separation region via turbulent diffusion, which is rather high in the flow around forward-facing steps because of the shock wave–boundary layer interaction. In the case of backward-facing steps, the shear layer moves away from the body surface, and turbulent diffusion is not high after interaction with the expansion wave. Therefore, the change in ω_0 has no effect on the flow behavior on the inclined face and in the region of the separation point. It follows from the analysis that this parameter has no significant effect on the turbulence parameters in the region of interaction of the boundary layer and expansion waves.

Conclusions. A supersonic turbulent flow around backward-facing steps with different face angles is numerically studied. The averaged Navier–Stokes equations supplemented by the Wilcox k–ω model of turbulence were used as a mathematical model. Two TVD schemes based on different methods of flux-vector splitting were used to approximate spatial derivatives of inviscid flows. The parameters of turbulent separated flows with a changed sequence of interaction were calculated by the algorithm used previously for calculating the flow around forward-facing steps. The agreement of numerical and experimental data was obtained for all available parameters including heat-transfer intensity.

The comparison showed that the scheme of splitting with respect to physical processes, which has a better resolution, has some special features in the case of the reverse sequence of shock wave–boundary layer interaction. For this reason, it is not possible to state definitely the applicability of this scheme for the above class of flows. The use of the scheme of [9] leads to the appearance of an expansion fan downstream of the interaction region, which is not observed in the experiment. The presence of these waves is rather probable from the physical viewpoint; however, their absence in the experiment may indicate their "scheme" origin. At the same time, the decrease in the experimental value of the heat-transfer coefficient in this region may be explained exactly by the presence of this expansion fan.

An additional study is also needed for the method of turbulence simulation in the case of interaction of the turbulent boundary layer with expansion waves. The two-parameter turbulence model proposed by Wilcox, which is used in the present work, predicts strong degeneration of turbulent oscillations in this region.

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